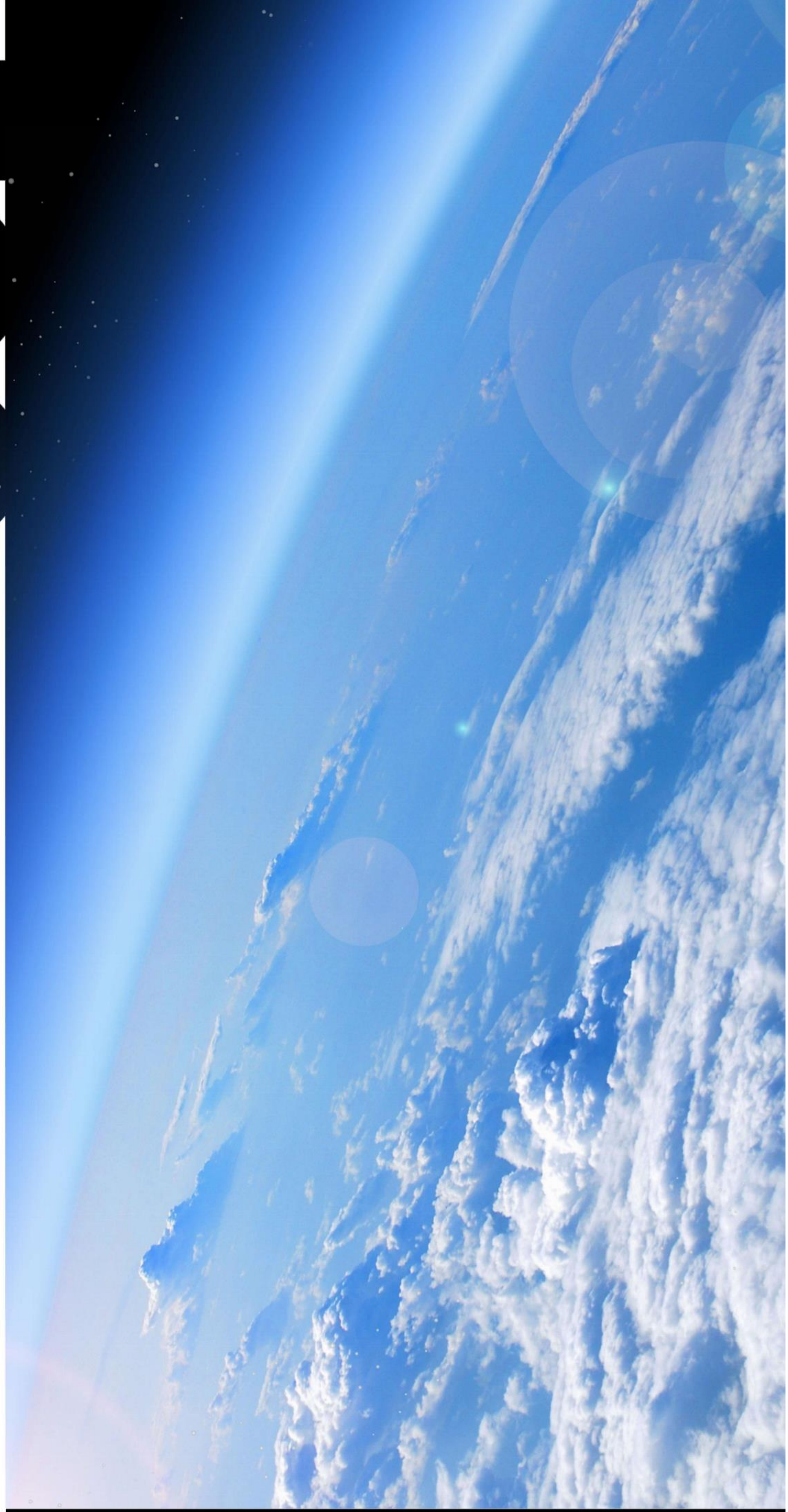


ENGFOOO3

Coursework 1

2022-2023

Information

Release Date:	17 Oct 2022
Submission Deadline:	1 Nov 2022 at Midday
Estimated Coursework Return:	Four weeks after submission
Topics Covered:	Topics 1 – 3
Expected Time on Task:	16 hours

Guidance for Submissions: Failure to follow this guidance might result in a penalty up to 10% in your marks.

I. Submit a single Word/LaTex/PDF document with models in order, from 1 to 5.

II. Do not write down your name, or student number, or any information that might help identifying you in any part of the coursework – including the file name.

III. Do not copy and paste the coursework questions into your submission – Simply rewrite information where necessary for the sake of your argument.

IV. Type your answers in Microsoft Word or any text editor, insert any relevant graphs or figures, and provide a description to any figures or tables in your document. All figures must be labelled, with their axes showing relevant parameters and units.

This coursework counts towards 20% of your final ENGF0003 grades and is made up of five questions, referred to herein as models. At the end, you are asked to write a short paragraph reflecting on your learning.

On Academic Integrity (Read more about it here)

Academic integrity means being transparent about our work.

- Research: You are encouraged to research books and the internet. You can also include and paraphrase any solution steps accessible in the literature and online content if you reference them.
- Acknowledge others: We are happy when you acknowledge someone else's work. You are encouraged to highlight if you found inspiration or part of your answers in a book, article or teaching resource. Read more about how to reference someone else's work here and how to avoid plagiarism here.
- Ask good questions to your peers: These include but are not limited to questions like "What is the best mathematical method for this question?", "Should I review any books/materials/videos?", "Which MATLAB function did you use for this problem?", "How was the structure of your MATLAB code?".
- Be helpful and ethical when answering questions from your peers: These include “I think that it would be helpful to review X video, Y page of the slides/notes”. “I found this good video online”, “I used X MATLAB function, structured that way”.
- Do not share and do not copy: We expect students not to share and not copy assessments from their peers, even if partially.
- Do not publish ENGF0003 assessment material: We expect students to not share ENGF0003 assessment materials at external online forums, tutoring, or "homework" help websites.

Students found in misconduct can receive a 0 mark in that assessment component and have a record of misconduct in their UCL student register. In some extreme cases, academic misconduct will result in the termination of your student status at UCL.

What are we assessing?

Since this is your first coursework, this paper will assess the following:

1. Your ability to understand and summarise a problem, describing important quantities and units. It is essential that you summarise the information that can be obtained from the problem brief and the reading list. Then, you should demonstrate how you can use this information to calculate a solution.

2. Your ability to learn from similar problems to those you are solving. The fundamental mathematical steps required to solve this coursework are exemplified in the reading list to various degrees. You should learn from them, analysing and explaining their methods, techniques, and concepts in your own words and referencing when necessary.

3. Your ability to communicate mathematics as applied to a real-life problem. Mathematics does not exist in a vacuum. We are not only assessing your ability to perform mathematical steps, but also your ability to communicate them clearly, in a coherent report. Remember that your paper should look more like a report from a junior engineer than a series of homework calculations.

Learning Outcomes Assessed

Model	Topic 2, Derivatives	Topic 3, Integrals
1 and 2	• Evaluating derivatives through the definition $\lim_{\to} \frac{(\quad)\quad(\,)}{}$; • Identifying the numerical convergence of derivatives • Applying derivatives for approximating lines • Modelling derivatives in physical phenomena through finite differences;	□ Not assessed in M1 and M2
3	• Recalling the concept of antiderivatives. • Discussing the relationship between derivatives and antiderivatives;	• Identifying definite integrals as the result of cumulative phenomena; • Discussing Riemann sums • Drawing analogies between mathematics concepts and real-life applications;

4 and 5	• Applying the chain rule to combine given expressions for ___ and ___; • Evaluating the effects of the direction of the □ axis on the sign of a derivative; • Reading and extracting data from figures; • Defining linear approximations via the concept of a gradient.	• Setting integration limits; • Evaluating definite integrals of expressions of the form □□ and □□□; • Contrasting and comparing mathematical models; • Plotting functions with MATLAB.

Background

In 2007, the Fédération Internationale de Football Association (FIFA) temporarily banned football matches at altitudes above 2.5 km from sea level, citing concerns that home teams would hold an “unfair” advantage over those not used to playing at high altitudes.

This ruling meant that countries like Bolivia, Ecuador, and Colombia would be prevented from hosting FIFA World Cup qualifiers and matches in their capitals. The ban lasted from May 2007 to May 2008. Later, in a 2008 paper to the Scandinavian Journal of Medicine & Science in Sports, US researchers proposed that altitude impacts football performance through two pathways:

(i) The rarefied oxygen in the air reduces maximal aerobic power.

(ii) The reduced air density facilitates high-velocity running but impairs sensorimotor skills[1].

As engineers, we can approach this problem from a physical-mathematical perspective by asking the question:

How does altitude affect the atmospheric pressure and the air that we breathe?

In this assessment, you are asked to create a mathematical model to understand the effects of altitude on the atmospheric pressure and density of air. This will enable researchers and football policymakers to understand the exact physical differences between playing sports at high and low altitudes and inform future dialogue around this topic.

Introduction

One of the most important properties of gases is that their pressure and density are related by the ideal gas law, given by:

$$p = \rho R T \qquad (1)$$

where p is the pressure [Pa], ρ is the density [kg m^{-3}], R is the universal gas contant R = 287.053 [J $kg^{-1}K^{-1}$] and T is the temperature of the gas [K]. The ideal gas law states that, in a closed volume, the gas pressure □, is proportional to the density of gas particles □ by a factor of □□.

To understand this law, let us imagine a small cube of air containing an arbitrary number of molecules, as shown in Figure 1. Notice that the left cube contains fewer molecules inside, having a density □ that creates a pressure □(□). However, if we squeeze all exterior particles inside the cube, the density increases to □ + Δ□, which results in a higher pressure □(□ + Δ□).

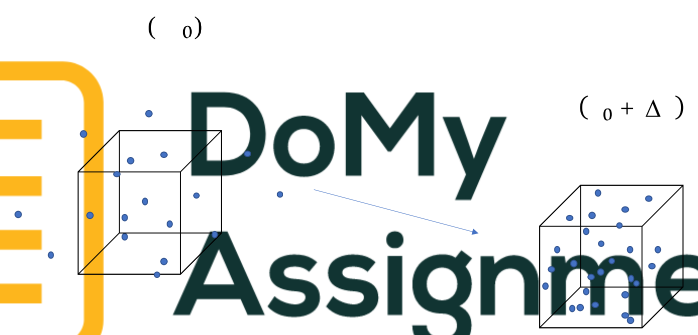


Figure 1.

Model 1 (15 marks)

Suppose that changing the density by Δ□ = □ − □ from a reference density □ , causes a particular variation in pressure Δ□ = □(□ + Δ□) − □(□). Apply the ideal gas law in

Equation 1 to find an expression for the ratio __. Start by Using Eq. 1 to evaluate □(□) and □(□ + Δ□), then work algebraically until you obtain an expression of the form __.

If at sea-level conditions the density of air is □ = 1.225 [kg m^{-3}] and □ = 288.15 K, apply the concept of a derivative to numerically evaluate □(□ + Δ□) for Δ□ = 0.5, 0.25, 0.1, 0.05 and 0.01 by constructing linear equations. Remember that a linear equation is of the form □(□) = □□ + □, where □ = __ and □ = □(0). Can you identify a mathematical process taking place as you reduce Δ□ from 0.5 to 0.01?

Solution:

Model 1

$$\Delta\square = \square\square$$

$$\Delta\square = \square(\square + \Delta\square) - \square(\square)$$

$$\Rightarrow \Delta\square = \square(\square + \square) - \square(\square)$$

$$\Rightarrow \Delta\square = \square(\square) - \square(\square) \rightarrow (\square)$$

Divide equation A by density difference;

$$\Rightarrow \frac{\Delta\square}{\Delta\square} = \frac{\square(\square) - \square(\square)}{\square - \square}$$

⇒ Given that;

$$\square = 1.225\ \text{kg/m}$$
$$\square = 288.15\ \text{k.}$$

$$\square = 8.314\text{J/mol} \cdot \text{k.}$$
$$\square = ??\ @(\Delta\square = 0.5 - 0.01)$$

Analysis:

$$\because\ □(□) = □□□.$$

$$\Rightarrow □(□) = □□□. \quad \text{and} \Rightarrow$$

$$□(□ + \Delta□) = □□(□ + \Delta□).$$

$$\Delta□ = 0.5;\ □(□ + \Delta□) = (8.314)(288.15)(1.225 + 0.5) \Rightarrow 4132.55□$$

$$\Delta□ = 0.25;\ □(□ + \Delta□) = (8.314)(288.15)(1.225 + 0.25) \Rightarrow 3533.62□$$

$$\Delta□ = 0.1;\ □(□ + \Delta□) = (8.314)(288.15)(1.225 + 0.1) \Rightarrow 3174.27□$$

$$\Delta□ = 0.05;\ □(□ + \Delta□) = (8.314)(288.15)(1.225 + 0.05) \Rightarrow 3054.27□$$

$$\Delta□ = 0.01;\ □(□ + \Delta□) = (8.314)(288.15)(1.225 + 0.01) \Rightarrow 2958.66□$$

Model 2 (15 marks)

Although very simple, the method of dividing space into very small cubes is an incredibly powerful approach. This is the foundation on which many sophisticated engineering calculations are built, such as finite elements or finite volume methods. Let us focus on a single cube filled with air molecules. This cube has length Δ□, width Δ□ and height Δ□, as shown in Figure 2.

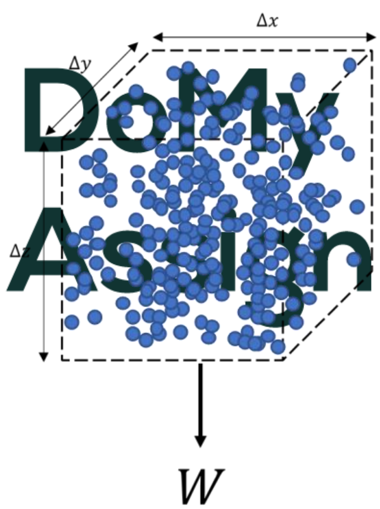


Figure 2.

Knowing that the weight of a body is given by □ = □□, where □ [kg] is its mass and □ is the acceleration due to gravity, find an expression for the pressure exerted by this volume on the bottom face of the cube. Then, use dimensional analysis to expand this idea toward finding an expression for the ratio ___.

Model 2:
we know that; $P \propto l$

$$\Rightarrow \quad P = \ell RT.$$

we also know that According to Pascal law pressures is exerted in all direction by the volume equally.
So to calculate the pressure at the bottom bocce of cube; we observe that;
$P_{\text{eQuation}} = P_{\text{by volume}} + P_{\text{by aright}} \cdot$
$\Rightarrow P_{\Theta b_0 + t_m} = P + \frac{W}{A} = \underline{\ell RT} + \frac{mg}{\Delta x \Delta y}$
$\Rightarrow P_{E\,\text{bottom}} = \frac{lRT}{} + \frac{lV)g}{\Delta x \Delta y} = \frac{lRT}{} + \frac{(\Delta + \Delta\Delta V \Delta z)g}{\Delta x \Delta g}$
$\Rightarrow P@$ bottom $= l(\underline{RT} + g\Delta z)$.
Now for $\frac{\Delta P}{\Delta z}$;

$$\because \Delta P \Rightarrow \Delta l(RT + g\Delta z).$$

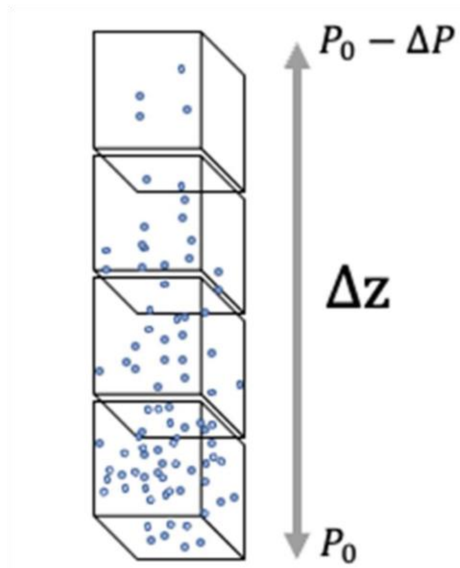
divide b/s by Δz;

$$\frac{\Delta P}{\Delta z} = \frac{\Delta l}{\left(\frac{RT}{\Delta z} + g\right).}$$

$$\Rightarrow \frac{\Delta P}{\Delta z} = (l - l_0)\left(\frac{RT}{\Delta z} + g\right).$$

Model 3 (20 marks)

If we imagine a column of air above an area Δ□Δ□, we can visualise in Figure 3 that the atmospheric pressure is the cumulative build-up of the weight of a column of air at a vertical distance Δ□ from the surface Δ□Δ□.



This makes the atmospheric pressure behave as a function of altitude: at low altitudes, there is a large column of air contributing to an increase in the pressure, whilst at higher altitudes, this column is smaller, and the pressure is consequently lower.

Figure 3.

In reference 2, you will find that:

$$\frac{□□}{□□} = -□□. \qquad (2)$$

Use your knowledge of calculus to show how we can use a Riemann sum to calculate and explain the cumulative effect of altitude on the atmospheric pressure. Discuss what are the limits of integration that should be used in this equation. You can support your argument with pictures and schematics.

Model 3:
Given that:

$$\frac{dP}{dz} = -\rho g. \Rightarrow dP = -lgdz$$
$$\Rightarrow P(z) = -lgdz.$$

Applying Riemann δum from z_0 to ${}_0z$;

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

$$\text{Here } f(x) = d(z) = -\lg dz.$$
$$x = \frac{\Delta z}{n}$$
$$\Rightarrow \Delta x = \frac{\Delta z}{n}$$
$$\Rightarrow (x_i)^* = a + (\Delta x)^* = z_0 + \left(\frac{z}{n}\right)i.$$
$$\Rightarrow f(x_i)^* = -\lg\left(z_0 + \left(\frac{\Delta z}{n}\right)i\right).$$

Substituting in Riemman Sum; we get:

$$\int_{z0}^{z} P(z)dz = \lim_{n\to\infty} \sum_{i=1}^{n} \left[-\lg\left(z + \left(\frac{\Delta z}{n}\right)i\right)\right]\frac{\Delta z}{n}.$$
$$\Rightarrow = \lim_{n\to\infty} -\lg \sum_{i=1}^{n} \left(\frac{z_0\Delta z}{n} + \left(\frac{\Delta z^2}{n^2}\right)i\right).$$
$$= \lim_{n\to\infty} -\lg \frac{z0\Delta z}{n} + \frac{\Delta z^2}{m^2}\sum_{i=1}^{n} i.$$
$$= \lim_{n\to\infty} -\lg \frac{z_0\Delta z}{n} + \frac{\Delta z^2}{n^2}\frac{(n)(n+1)}{2}.$$

$\Rightarrow$ Apply $\lim\limits_{n\to\infty}$

$$\Rightarrow \int_{z0} P(z)dz = -\lg z_0\Delta z + \frac{\Delta z^2}{z} \quad \text{Lower limit } z_0$$

Model 4 (20 marks)

There are two fundamental ways in which the atmospheric pressure changes.

The first, is modelled by the ideal gas law and describes the effects of pressure and temperature on the density. The second, is that the air pressure varies with altitude at a rate — due to the weight of the column of air above a certain reference area $\Delta□\Delta□$, as shown in Equation 2.

Combine these two ideas and use the working in Model 3 to show that

$$□□ = -\frac{□}{□}\,□□\,.\,\frac{}{□□} \qquad (3)$$

Then, employ the chain rule to obtain an expression for —.

Solution:

Model u:
Analysis:
Ideal gas law: $P = \ell RT \longrightarrow (1)$
Pressure Rate: $\frac{dp}{dz} = -lg \rightarrow 2$
Now from equation 1, we get

$$l = \frac{P}{RT} \rightarrow \ (3).$$

Put equation 3 is equation (2):

$$\Rightarrow \frac{dP}{dz} = -\left(\frac{P}{RT}\right)g.$$

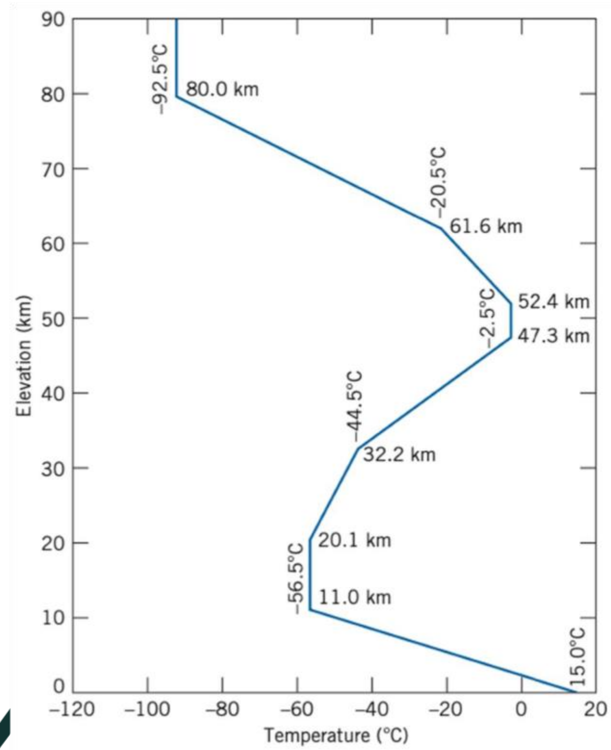
$$\Rightarrow \frac{dP}{P} = -\frac{g}{RT}dz.$$

Hence proved.

Model 5 (20 marks)

Equation 2 correlates variations in pressure □□ to variations in altitude □□ via the ratio □□. If we use the ideal gas law to approximate □ as a function of temperature, we obtain Equation 3, that relates □□ and □□ via the gas constant □ and the temperature □, that is also assumed to be constant in Equation 1. However, experimental measurements plotted in Figure 4 show that the altitude also affects the atmosphere's temperature □.

Figure 4.



Use the data in Figure 4 to find an expression for the temperature □ as a function of the elevation (altitude) □ where □ is between 0 and 10 km.

Employ this expression to solve Equations 2 and 3. Then, contrast and compare the results that you find for the case of a constant temperature □ and an altitude-dependent temperature □(□) when solving Equation 3.

Finally, use MATLAB to produce a plot for the expressions obtained as a function of altitude and comment on your results.

Model 5:

Analysis:

Frown Figure' 4 we get;

$$(T_1, z_1) = (x_1, y_1) = (20, 0)$$
$$(T_2, z_2) = (x_2, y_2) = (-56.5, 10)$$

Now Applying two point form;

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\Rightarrow \frac{Z-Z_1}{T-T_1} = \frac{Z_2-Z_1}{T_2-T_1}$

$\Rightarrow z = z_1 + \left(\frac{z_2-z_1}{T_2-T_1}\right)(T - T_1)$

$\Rightarrow z = \frac{0+10-0}{(-56.5-20)}(T - 20).$

$\Rightarrow Z = -0.1307T + 2.614$

$\Rightarrow z(T) = -0.1307T + 2.614$

$T(z) = 2.614 - 7.6511(z)$

Put $T(2)$ in eq 3 ;

$$\frac{dP}{P} = \frac{gdz}{R}\left(\frac{1}{T}\right) = \frac{g}{R}\left(\frac{1}{2.614 - 7.651z}\right)dz$$

$\Rightarrow$ Apply integral on b/6:

$$\int_{P_0}^{p} \frac{dP}{p} = \frac{g}{R}\int_0^0 \left(\frac{1}{-7.651z + 2.614}\right)dz.$$

Here the limits in L.H.S are from Zo to Z.

$$\Rightarrow \ln P - \ln P_0 = \frac{\theta}{R}\left|\frac{\ln(-7.651z + 2.614)}{-7.651}\right|$$

$$\Rightarrow \ln \frac{P}{P_0} = \frac{g}{R}\left|\ln \frac{(-7.651z + 2.614)}{-7.651}\right|$$

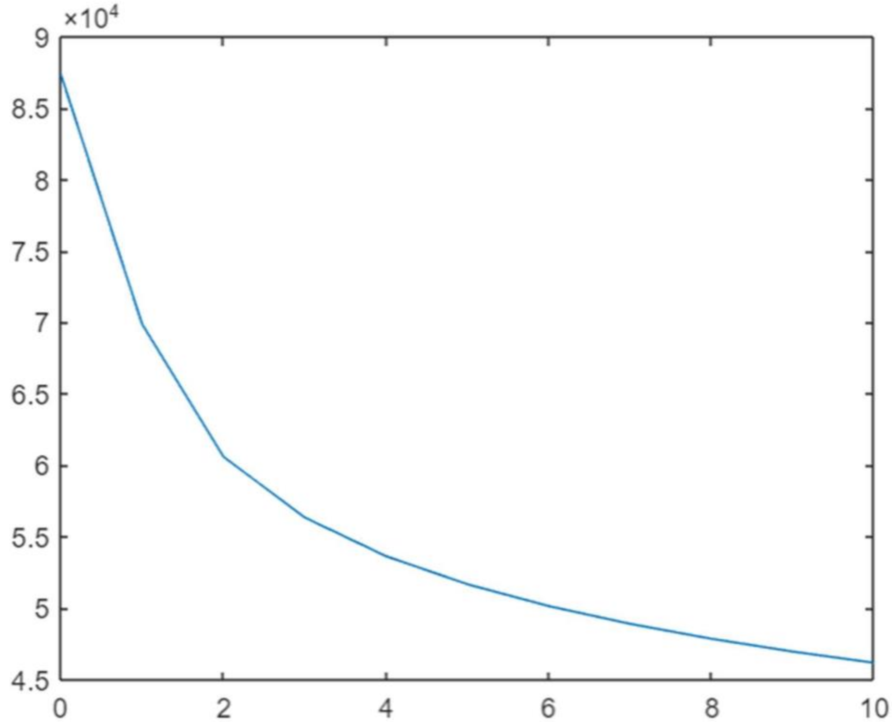
$$\Rightarrow \frac{P}{P_0} = \exp\left(\frac{g}{R}\frac{\ln(-7.6512 + 2.614)}{-7.651}\right)$$

COMPARISION:

By comparing both the equations we came to know that by accumulating temperature changes with respect to change in altitude there is a significant change in pressure variation

MATLAB CODE:

```
% Matlab Code
% Altitude dependent Pressure Variation clc
Po = 101325  g =
9.81 R = 8.314
% z is the altitude z =
0:1:10
Y = -7.65*z + 2.614
X = (g/R)*(log(Y)/-7.65)
P = Po*exp(X)
 plot(z,P)
```


×10^4
9
8.5
8
7.5
7
6.5
6
5.5
5
4.5
0
2
4
6
8
10

Summary (10 marks)

This study involves the mathematical modelling of Pressure and altitude of atmospheric air and the air we breathe. There are total 5 models of pressure variation versus altitude and pressure versus temperature collectively. Model 1 is about the pressure variation with respect to density variation of air at constant temperature _, which is the best example of isothermal process in many thermodynamic systems. Model 2 is the modelling of pressure exerted on the bottom face of a cube by the air with respect to the height of the cube _.

This model is normally used in container designing for engineering applications. Model 3 is the more modified form of the model 3 in which Riemann's Sum approach is applied for the pressure measurement with the change in differential height in a column. Model 5 is the representation of effects of pressure and temperature on the density. Secondly the air pressure varies with altitude at a rate _ due to weight of the column of the air above a certain height reference area Δ□Δ□. In Model 5 we have modelled the pressure variation of atmospheric air from 0 to 10 km. First we build the expression for temperature as a function of altitude z from figure 4, then by substituting the temperature expression in equation 3 and simplifying, we get the expression for atmospheric pressure.

To conclude, we can say that the most appropriate model of atmospheric pressure variation is the Model 5. And the most appropriate model of air we breathe is Model 3 which includes Riemann's Sum approach.

Reading List

1. (Optional) Levine BD, Stray-Gundersen J, Mehta RD. Effect of altitude on football performance. Scand J Med Sci Sports. 2008 Aug;18 Suppl 1:76-84. doi: 10.1111/j.1600-0838.2008.00835.x. PMID: 18665955. [LINK]
2. (Essential) Pritchard, P. J., & Mitchell, J. W. (2015). Fox and McDonald's introduction to fluid mechanics (8th ed.). John Wiley & Sons. Chapters 1 to 3. Some of this content will be beyond your mathematical abilities, so you should focus exclusively on sections 1.1, 1.4, 1.6, 2.1, 3.2 and 3.3. [LINK]
3. (Essential) ENGF0003 Moodle, Workshop and Computational Modelling Materials, Weeks 1 to 3.
4. (Optional) Rodrigues, Diego & Arnold, Francisco. (2022). Analyzing Atmospheric Pressure Variations in Time and Height: a Didactic Proposal Employing a Smartphone Barometer. Revista Brasileira de Ensino de Física. 44. 10.1590/1806-9126-rbef-2021-0422. [LINK]
5. (Optional) Monteiro, Martín & Vogt, Patrik & Stari, Cecilia & Cabeza, Cecilia & Martí, Arturo. (2015). Exploring the atmosphere using smartphones. The Physics Teacher. 54. 10.1119/1.4947163. [LINK]

Rubric for technical execution (60% of marks) Rubric for reasoning and

	Poor			Good		Excellent	
	20%	40%	50%	60%	70%	85%	100%
Recall and referencing of formulas	Incorrect		Correct recall with inappropriate, missing or unclear referencing. There are no/poor explanations of formulas, parameters and terms.			Correct recall and appropriate referencing, formulas are explained in detail.	
Technical execution	Multiple/major errors		Few/minor errors		No errors, difficult to follow	No errors, succinct, coherent, and easy to follow	
Final Answer	Missing		Incorrect but acceptable		Correct mathematics, but poor/absent reasoning	Comprehensive answer, with correct mathematics and appropriate reasoning	

engineering skills (40% of marks)

	Poor			Good	Excellent		
	20%	40%	50%	60%	70%	85%	100%
Use of units	Absent		Minor errors, omissions, and inconsistencies		Effective and consistent throughout the script		Insightful use of units followed by appropriate discussion
Use of graphs or schematics	Absent		Incorrect or missing titles, labels, legends, descriptions		Appropriate, minor flaws in style, visibility, or relevance		Outstanding
Comment on assumptions and limitations	Absent		Limited comment, failing to reference the reading list		Appropriate comment, references the reading list		Insightful comment, references and expands upon the reading list
Validates answer or methods?	No						Yes